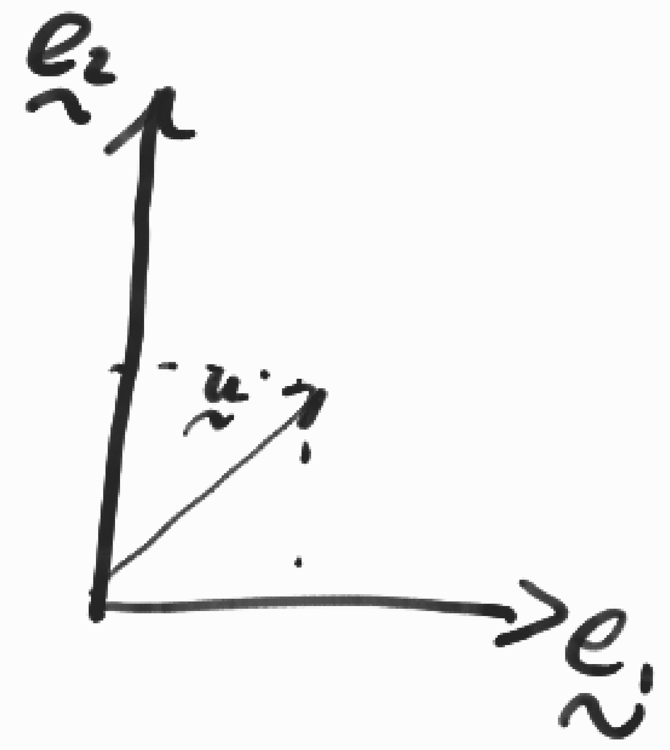
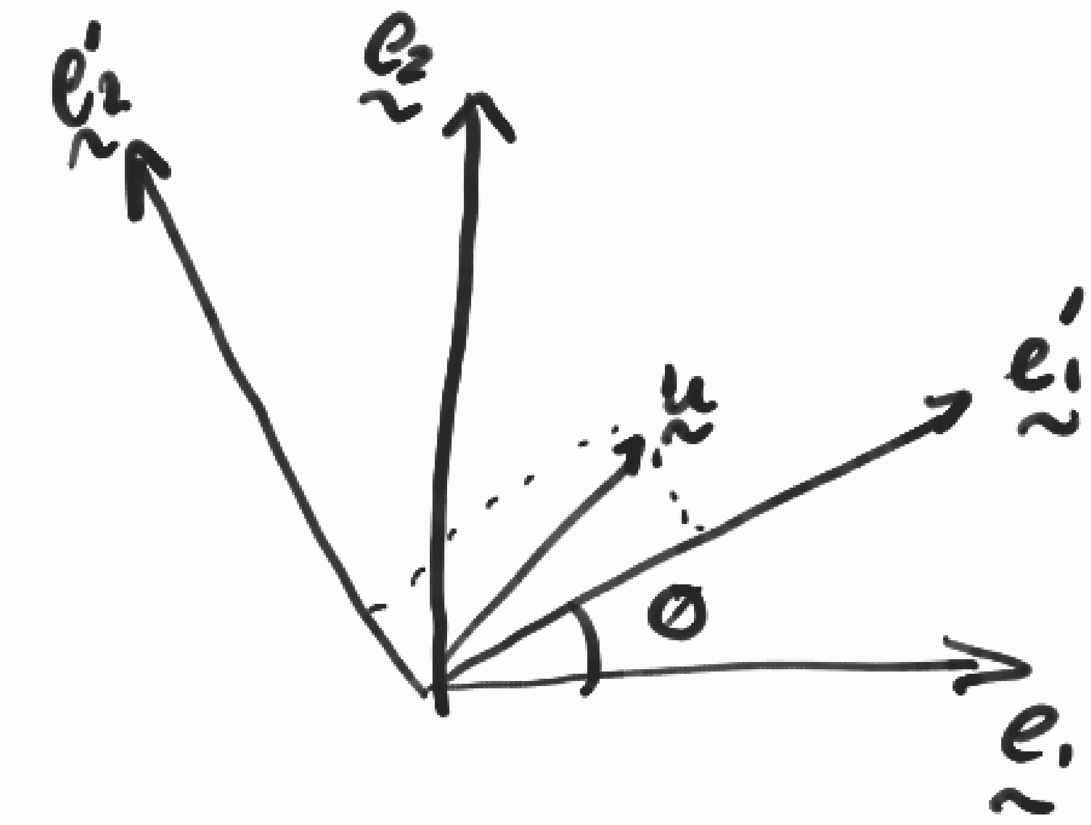
2. COORDINATE TRANSFORMATION





$\{\underset{\sim}{e}_i'\}$ IS A ROTATION OF BASIS SET $\{\underset{\sim}{e}_i\}$ BY ANGLE θ.

$$\underset{\sim}{u} = u_1 \underset{\sim}{e}_1 + u_2 \underset{\sim}{e}_2 = u_1' \underset{\sim}{e}_1' + u_2' \underset{\sim}{e}_2' \qquad (2.1)$$

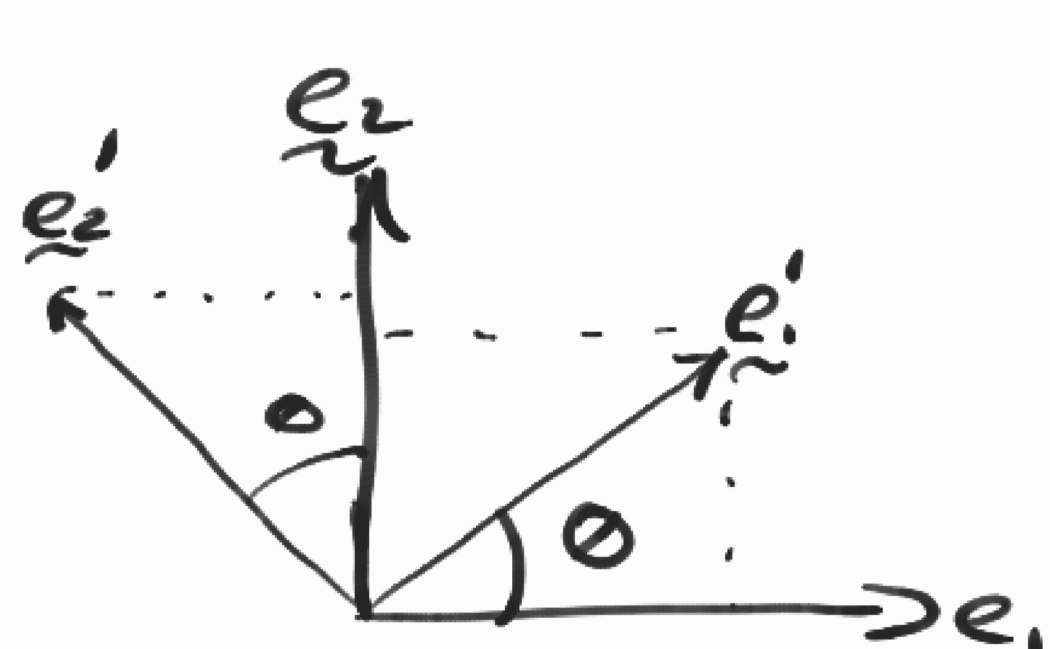
So $\underset{\sim}{u}$ IS THE SAME VECTOR REGARDLESS OF COOR. SYS. BUT, IT IS DESCRIBED IN TERMS OF DIFF COMPONENTS. I.e., IN GENERAL $u_i \neq u_i'$

So HOW ARE u_i and u_i' RELATED?

$$u_1' = \underset{\sim}{e}_1' \cdot \underset{\sim}{u} = \underset{\sim}{e}_1' \cdot (u_1 \underset{\sim}{e}_1 + u_2 \underset{\sim}{e}_2)$$
$$= u_1 (\underset{\sim}{e}_1' \cdot \underset{\sim}{e}_1) + u_2 (\underset{\sim}{e}_1' \cdot \underset{\sim}{e}_2)$$

SIMILARLY, $$u_2' = u_1 \underset{\sim}{e}_2' \cdot \underset{\sim}{e}_1 + u_2 \underset{\sim}{e}_2' \cdot \underset{\sim}{e}_2 \qquad (2.2)$$

HOW DO WE SIMPLIFY FURTHER FROM $\underset{\sim}{e}_i' \cdot \underset{\sim}{e}_j$? BY WRITING $\underset{\sim}{e}_i'$ IN TERMS OF $\underset{\sim}{e}_j$



(2.3)

$$\underset{\sim}{e}_1' = \cos\theta\, \underset{\sim}{e}_1 + \sin\theta\, \underset{\sim}{e}_2$$
$$\underset{\sim}{e}_2' = -\sin\theta\, \underset{\sim}{e}_1 + \cos\theta\, \underset{\sim}{e}_2$$

Plug 2.3 into 2.2 $\Rightarrow$ u_i' in terms of u_i

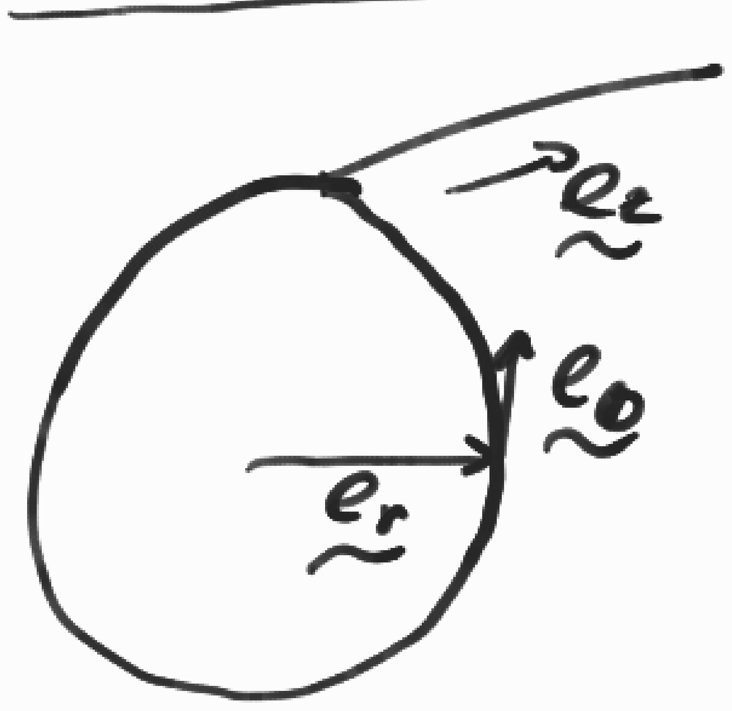
$$u_1' = u_1(\underset{\sim}{e}_1' \cdot \underset{\sim}{e}_1) + u_2(\underset{\sim}{e}_1' \cdot \underset{\sim}{e}_2)$$
$$= u_1 \cos\theta + u_2 \sin\theta$$
$$u_2' = u_1(\underset{\sim}{e}_2' \cdot \underset{\sim}{e}_1) + u_2(\underset{\sim}{e}_2' \cdot \underset{\sim}{e}_2)$$
$$= u_1(-\sin\theta) + u_2(\cos\theta)$$

$$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

↳ Transformation matrix

$\underset{\sim}{e}_i'$ and $\underset{\sim}{e}_i$ are both Cartesian coord. sys.
But for some problems, other coor sys make more sense

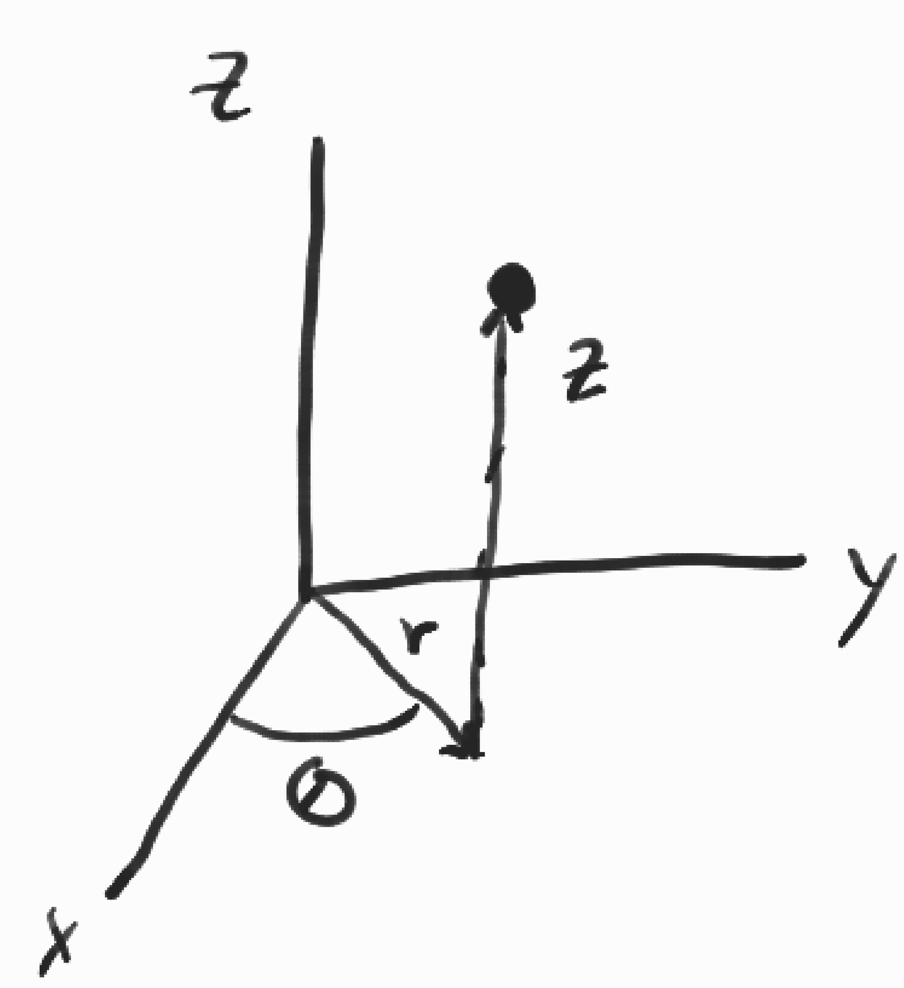
Polar / Cylindrical



$\underset{\sim}{e}_r$

$\underset{\sim}{e}_\theta$

$\underset{\sim}{e}_z$



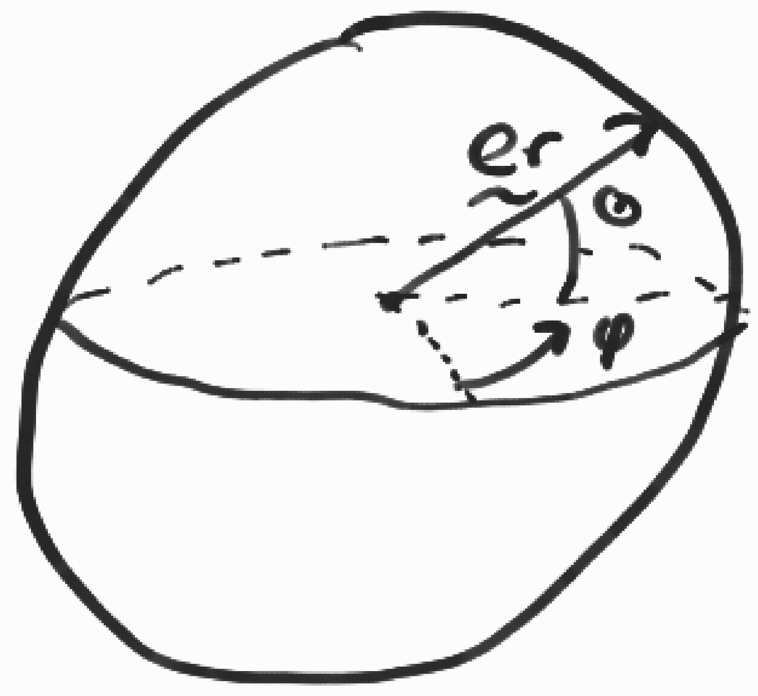
Examples

Bone

Vessels

trachea

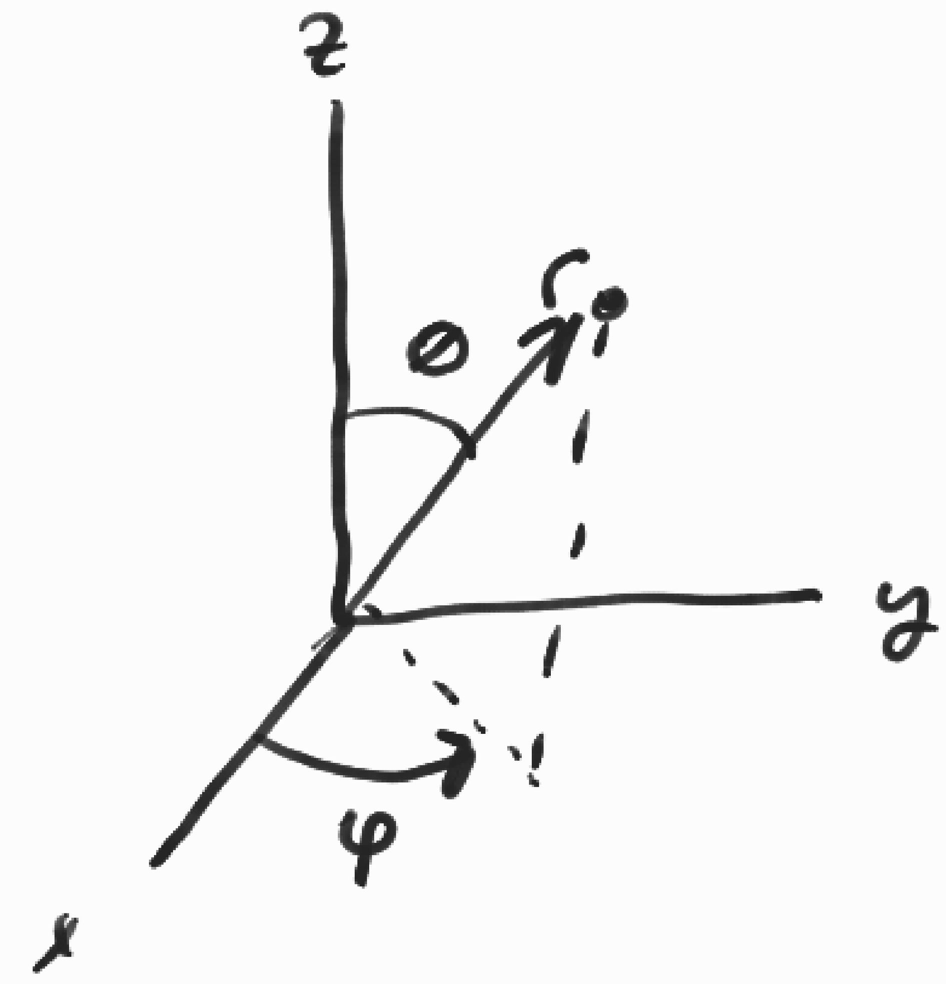
SPHERICAL



$\underset{\sim}{e_r}$

$\underset{\sim}{e_\theta}$

$\underset{\sim}{e_\varphi}$

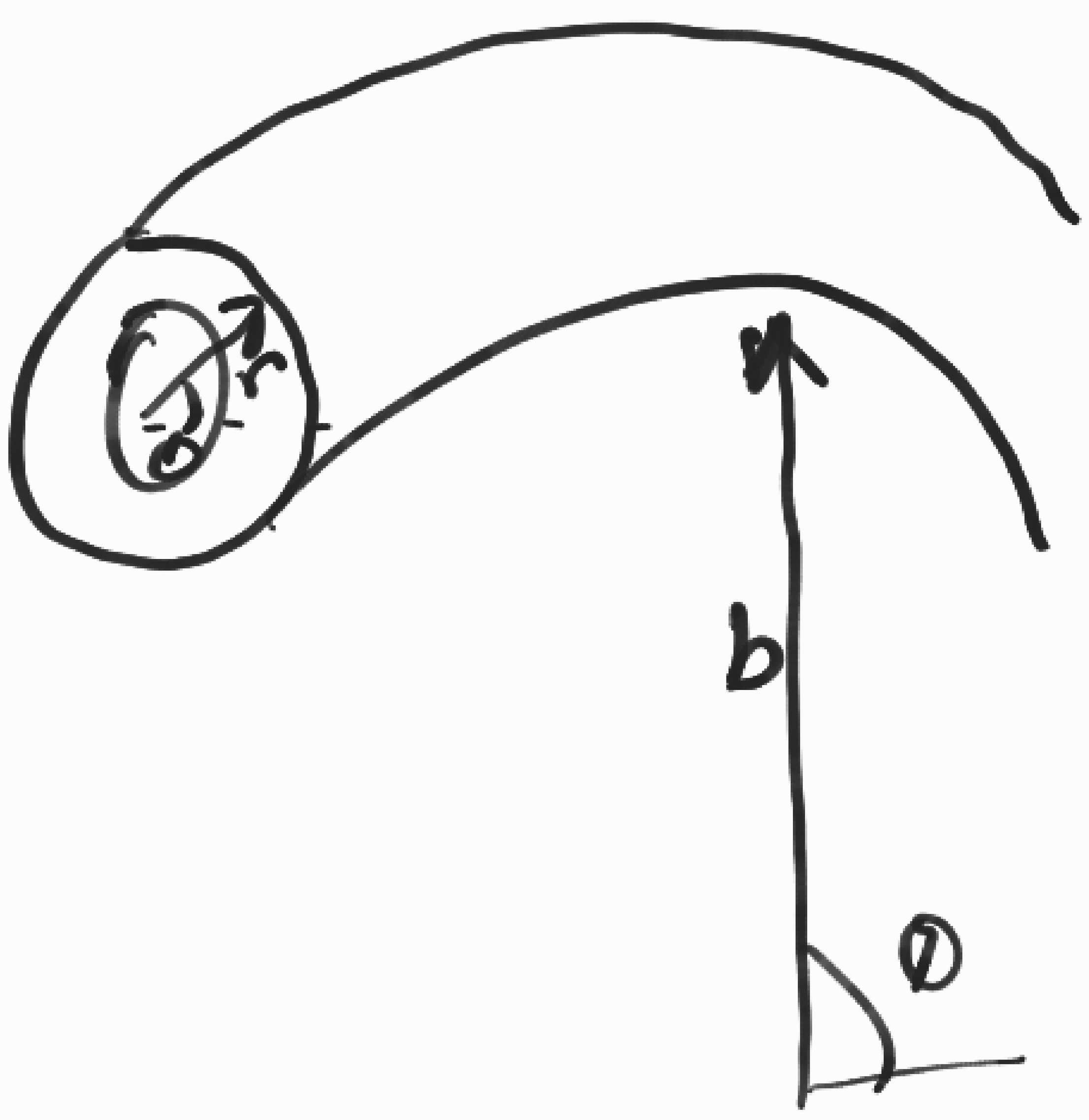


eyeball

alveoli

cell / nucleus

TOROIDAL

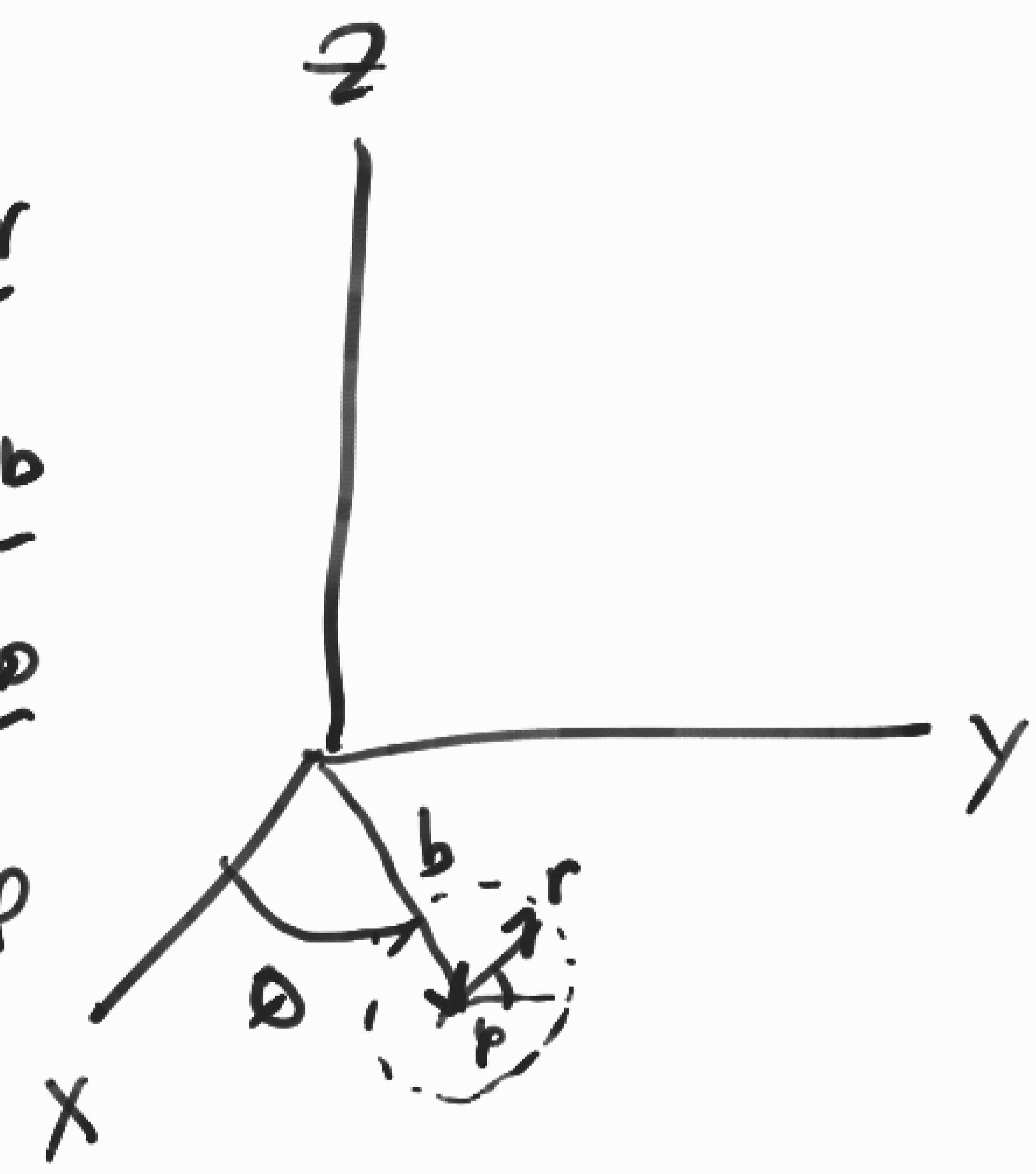


$\underset{\sim}{e_r}$

$\underset{\sim}{e_b}$

$\underset{\sim}{e_\theta}$

$\underset{\sim}{e_\varphi}$



Intestines

glomeruli